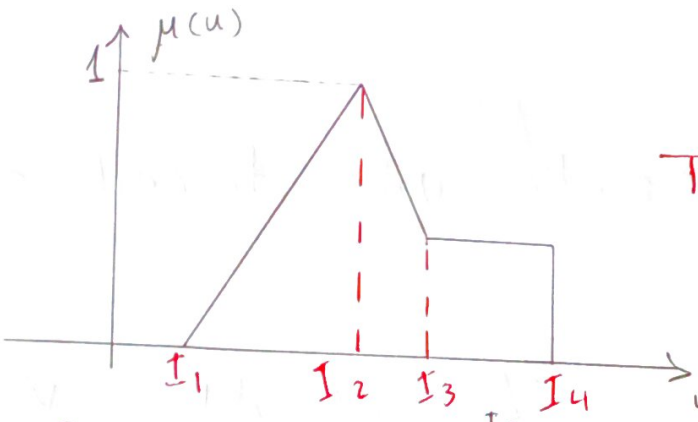
محاضرة 5 د. عرفة الثلاثاء 26/10/2016

* Common methods of difuzzication:

1. Center of Gravity Method (COG)
   ≡ center of area method (COA)
   ≡ The centroid method

$$u^{crisp} = \frac{\int \mu(u).u\,du}{\int \mu(u)}$$

Example:



The overall fuzzy o/p

$$u^{crisp} = \frac{\int \mu(u).u\,du}{\int \mu(u)} = \frac{\int_{I_1}^{I_2} \mu(u).u\,du + \int_{I_2}^{I_3} \mu(u).u\,du + \int_{I_3}^{I_4} \mu(u).u\,du}{\int_{I_1}^{I_2} \mu(u)\,du + \int_{I_2}^{I_3} \mu(u)\,du + \int_{I_3}^{I_4} \mu(u)\,du}$$

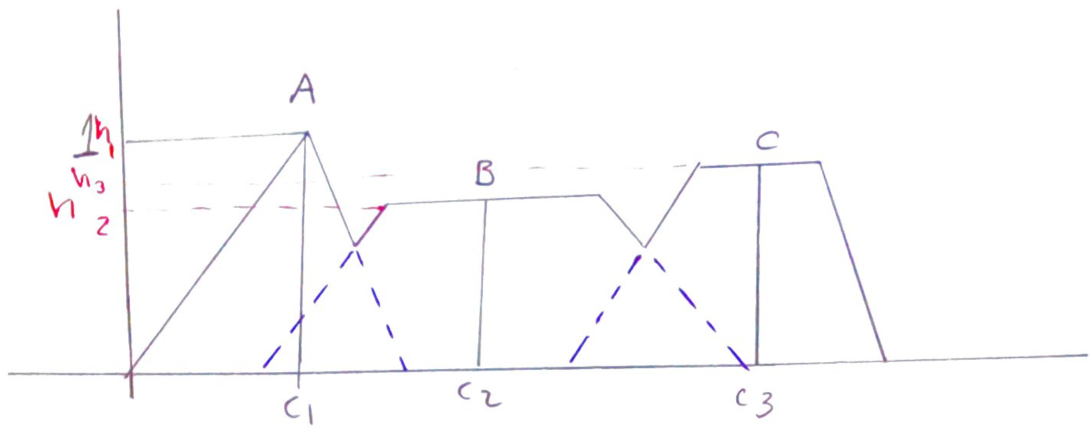
THE MOST ACCURATE METHOD

if eqns are hard to obtain, we use discrete method

$$\mu^{crisp} = \frac{\sum_{i=1}^{\tilde{N}} \mu(u_i).u_i}{\sum_{i=1}^{\tilde{N}} \mu(u_i)}$$ ← discrete (COA)

2 Max-Mean Membership method:

Example



$$U^{crisp} = \frac{c_1 + c_2 + c_3}{3}$$

$$\Rightarrow U^{crisp} = \frac{\sum_{k=1}^{N} c_k}{N}$$

c_k is the corresponding value to peaks of o/p fuzzy sets.

3 Weighted Average Method: لما يكون الشكل فيه تماثل

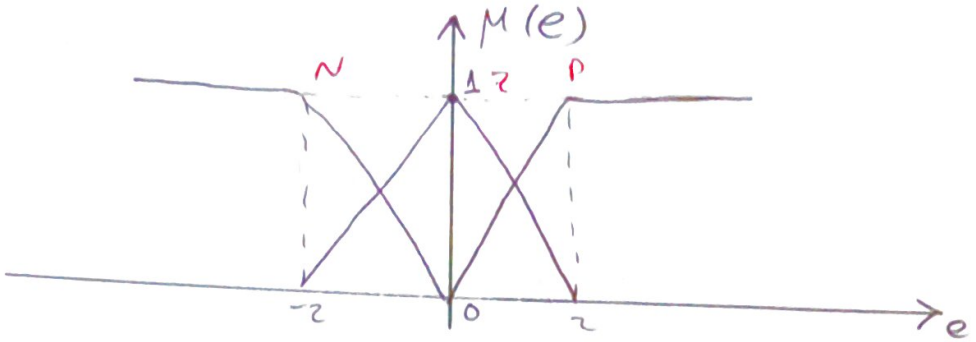
$$U^{crisp} = \frac{\sum_{i=1}^{N} \mu(u_i) . u_i}{\sum_{i=1}^{N} \mu(u_i)}$$

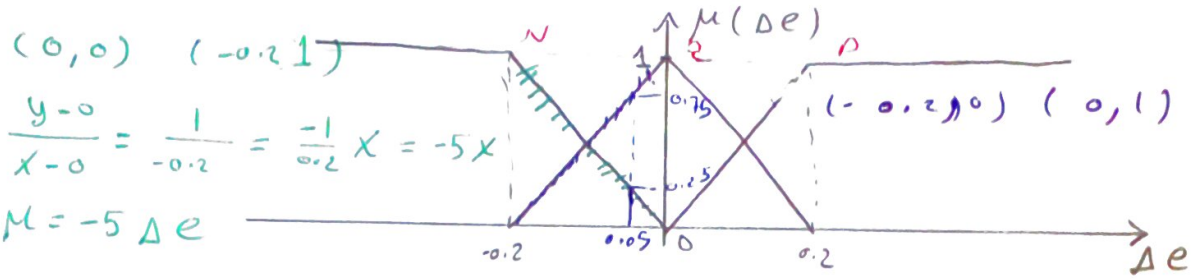
where u_i is the symmetric point to the fuzzy set o/p form.

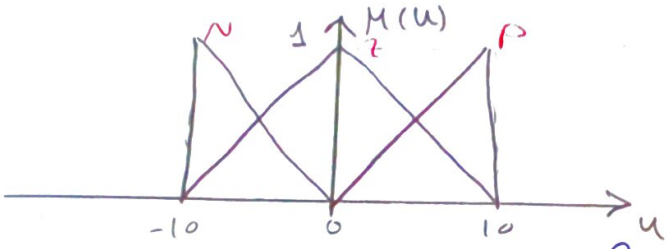
For the previous figure

$$U^{crisp} = \frac{c_1 h_1 + c_2 h_2 + c_3 h_3}{h_1 + h_2 + h_3}$$

Example







Find the crisp output in the following cases

① $e = -2$ & $\Delta e = -0.05$

② $e = 1.5$ & $\Delta e = -0.15$

③ $e = 2$ & $\Delta e = 0.2$

④ $e = 1$ & $\Delta e = 0.1$

e \ Δe	N	Z	P
N	P	P	Z
Z	P	Z	N
P	Z	N	N

fired rules

① $e = -2$ & $\Delta e = -0.05$

$e = -2 \rightarrow N$ with $\mu = 1$ → e is N

$\Delta e = -0.05 \rightarrow N$ with $\mu = (0.25)$ ← صح الرسم → Δe is N

$\rightarrow Z$ with $\mu = (0.75)$ ← صح الرسم → Δe is Z

Premise terms

[2] the fired rules (Which rules are on)

Check the previous table

R_1: if e is N and Δe is N1 then u is P

R_2: if e is N and Δe is Z1 then u is P

estimated terms

[3] Strength of fired rules

$$\mu P_1 = \min\{\mu_N(e=-2), \mu_N(\Delta e = -0.05)\}$$

$$= \min\{1, 0.25\} = 0.25$$

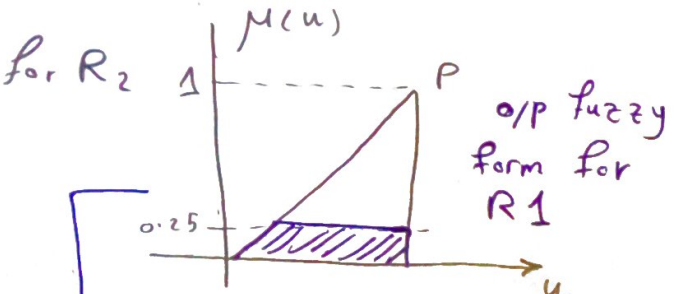
$$\mu P_2 = \min\{\mu_N(e=-2), \mu_Z(\Delta e = -0.05)\}$$

$$= \min\{1, 0.75) = 0.75\} = 0.75$$

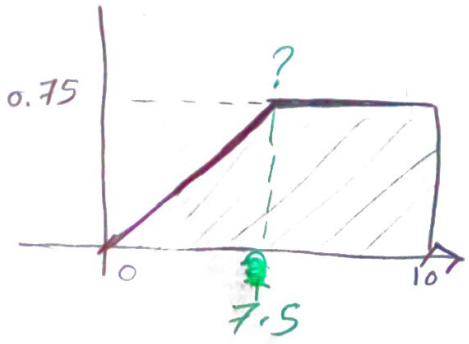
[4] The fuzzy sets output forms

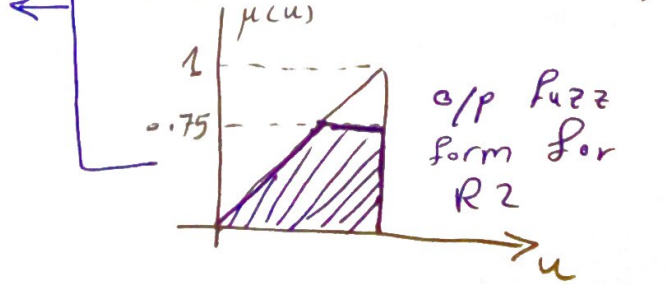
* $\mu_P(u) = \min\{\mu_P(u), 0.25\}$ → for R1

* $\mu_P(u) = \min\{\mu_P(u), 0.75)$ → for R_2



(5) overall output of fuzzy set





6 defuzzification using (COG) method :—

line eqn = $0.1u$

$\rightarrow 0.75 = 0.1u \Rightarrow u = 7.5$

$$u^{crisp} = \frac{\int_0^{7.5} \mu(u).u\,du + \int_{7.5}^{10} \mu(u).u\,du}{\int_0^{7.5} \mu(u)\,du + \int_{7.5}^{10} \mu(u)\,du} = \frac{I_1}{I_2}$$

$$I_1 = \int_0^{7.5} (0.1u)u\,du + \int_{7.5}^{10} 0.75u\,du = \left[\frac{0.1u^3}{3}\right]_0^{7.5} + \left[0.75\frac{u^2}{2}\right]_{7.5}^{10} = 30.468$$

$$I_2 = \int_0^{7.5} 0.1u\,du + \int_{7.5}^{10} 0.75\,du = \left[\frac{0.1u^2}{2}\right]_0^{7.5} + [0.75u]_{7.5}^{10} = 4.6875$$

$$u^{crisp} = \frac{I_1}{I_2} = 6.5$$

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2 $e = 1.5$ & $\Delta e = -0.15$

1 fuzzification :

$e = 1.5$ → Z with $\mu = 0.25$ → e is Z

→ P with $\mu = 0.75$ → e is P

$\Delta e = 0.15$ → N with $\mu = 0.75$ → Δe is N

→ Z with $\mu = 0.25$ → Δe is Z

[2] Fired rules

R_1: if e is Z and Δe is N then u is P

R_2: if e is P and Δe is N then u is Z

R_3: if e is Z and Δe is Z then u is Z

R_4: if e is P and Δe is Z then u is P

[3] strength of fired rules

$$\mu_{P_1} = \min\{0.25, 0.75\} = 0.25$$

$$\mu_{P_2} = \min\{0.75, 0.75\} = 0.75$$

$$\mu_{P_3} = \min\{0.25, 0.25\} = 0.25$$

$$\mu_{P_4} = \min\{0.75, 0.75\} = 0.25$$

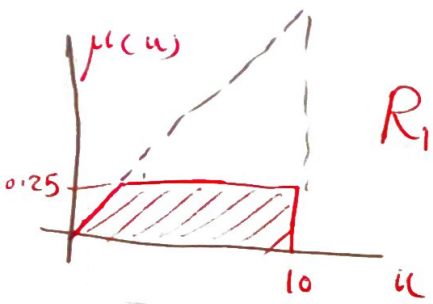
[4] forms of fuzzy sets o/p

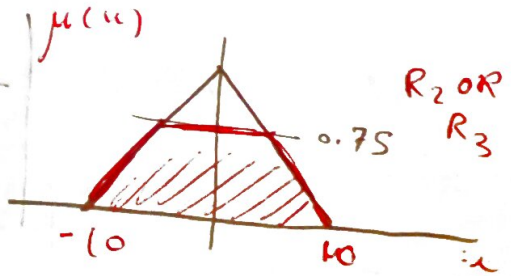
$$\mu_P(u) = \min\{\mu_P(u), 0.25\} \rightarrow R_1$$

$$\mu_Z(u) = \min\{\mu_Z(u), 0.75\} \rightarrow R_2$$

$$\mu_Z(u) = \min\{\mu_Z(u), 0.25\} \rightarrow R_3$$

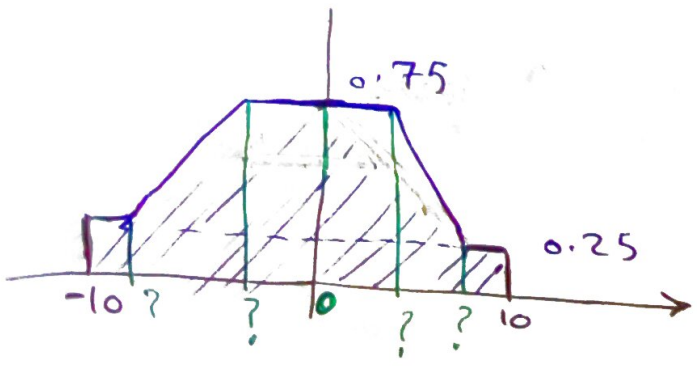
$$\mu_N(u) = \min\{\mu_N(u), 0.25\} \rightarrow R_4$$

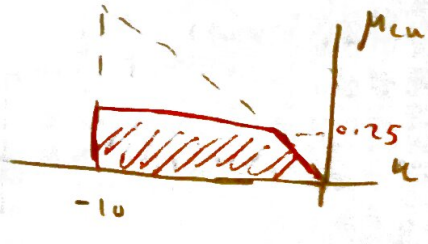




[5] overall fuzzy output

Aggregation (OR operation)

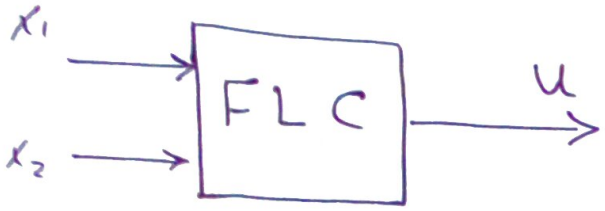


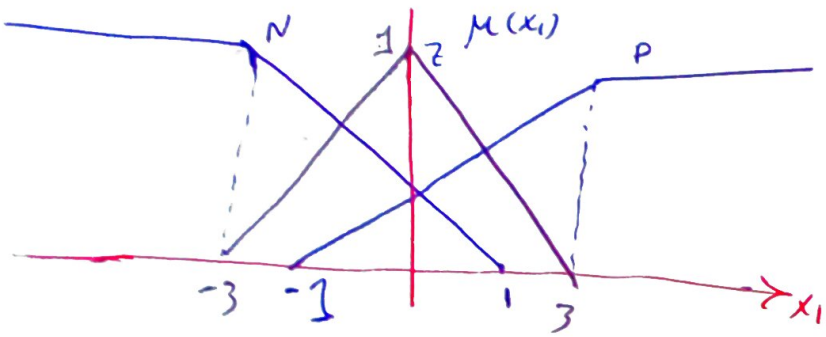


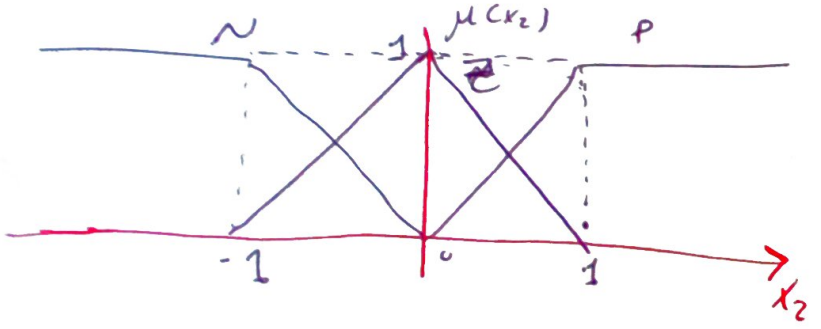
* Because the shape is symmetrical

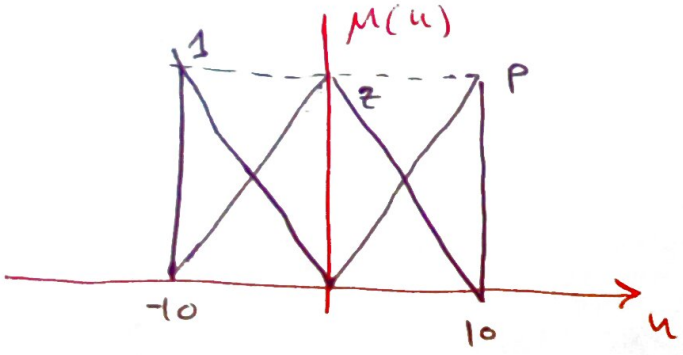
$u^{crisp} = 0$

REPORT









find the crisp o/p if $x_1 = 0.5$; $x_2 = 0$ for same table of rules in previous example using COG method